SELF-CONSISTENT PROBLEM ABOUT ELECTRIC FIELDS PRODUCED IN AIR BY A PULSE OF GAMMA QUANTA

M. F. Ivanov, A. A. Solov'ev, and V. A. Terekhin

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The problem of the radial electric field excited in air by an instantaneous point source of gamma quanta is considered. This problem was solved in [1-3] under the assumption that the Compton electron currents originating during scattering of the gamma quanta are given. Such an approximation is valid if the influence of the originating electric field on the Compton electron motion is neglected. The dimensionless parameter characterizing the influence of the electric field is $\alpha = e\varepsilon l/W$ (ε is the characteristic magnitude of the electric field, and l and W are the path and kinetic energy of the Compton electron, $W \sim 1$ MeV). For $\alpha \ll 1$ deceleration of the electrons by the electric field can be neglected and the model proposed for Compton currents [1] is used to determine the field.

A model applicable to the description of radial electric fields for $\alpha \gg 1$ is proposed in this paper. In particular, this condition is satisfied at kilometer distances from a gamma source with a total quantum yield of $N \geqslant 10^{23}$ [1-3] for surrounding air densities of $\rho \leqslant 10^{-4}$ g/cm^3. In this case, deceleration of the Compton electrons in the electric field is more substantial than the ionization losses. Indeed, $\varepsilon \sim \sqrt{8\pi n_k W}$, $\alpha \geqslant 10$. Hence, to describe the Compton electron motion a kinetic equation with a self-consistent field, in whose right side a source describing the production of Compton electrons behind the gamma-quanta front [4] is added, can be used.

The number of Compton electrons originating per unit time in unit volume of a phase space $(\mathbf{r}, \mathbf{p})$ at a distance r from a source is

$$S(r, p) = n_k \delta(p - p_0)\,\delta\left(t - \frac{r}{c}\right),$$

where $n_k = \frac{Ne^{-r/\lambda}}{4\pi r^2 \lambda}$ is the concentration of Compton electrons at a distance r from the source, N is the total number of gamma quanta, and λ is the path of the γ quanta.

It is assumed that all the electrons are produced with the identical initial momentum $p_0 = \sqrt{\frac{W(W + 2mc^2)}{c_2}}$, directed along the radius. Such a model approximation for S is legitimate, since the mean cosine of the angle between the initial direction of electron motion and the direction of the gamma-quanta flux is close to one for a gamma-quantum energy ~ 1 MeV [2].

Henceforth, such distances are considered at which the displacements of the Compton electrons are small compared to the path of the gamma quanta and the distance from the source. In this case the gamma flux can be considered planar, and the originating electric field is locally homogeneous. Hence, all the functions are considered dependent only on $\tau - (r/c)$. The kinetic equation for the electron distribution function f and the Maxwell equation for the electric field E are written in (τ, p) variables as

$$\left(1 - \frac{v}{c}\right)\frac{\partial f}{\partial \tau} - eE(\tau)\frac{\partial f}{\partial p} = n_k \delta(p - p_0)\,\delta(\tau); \quad \frac{dE}{d\tau} = 4\pi e \int v f dp.$$

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Using the method of solution from [4], an ordinary differential equation

$$\frac{d^2\Phi}{d\tau^2} = -4\pi j(\Phi) \qquad (1)$$

can be obtained to determine the field "potential" $\Phi(\tau) = \int_0^\tau E(\xi)\,d\xi$.

The current density of the Compton electrons j depends on Φ as follows:

$$j(\Phi) = \frac{1}{2} en_k c \frac{\left(\sqrt{p_0^2 + m^2c^2} - p_0 + e\Phi\right)^2 - m^2c^2}{\left(\sqrt{p_0^2 + m^2c^2} - p_0 + e\Phi\right)^2}.$$

Solving (1), we obtain

$$\tau = \frac{2}{\omega_p\sqrt{\beta}} \mathrm{E}\left(\arcsin\sqrt{\frac{e\Phi/mc}{(e\Phi/mc + \beta)(1-\beta^2)}};\ \sqrt{1-\beta^2}\right) - 2\sqrt{\frac{e\Phi/mc\,[1-\beta^2-\beta e\Phi/mc]}{\beta(\beta + e\Phi/mc)}};$$

$$\left(\beta = \sqrt{1 + p_0^2/m^2c^2} - p_0/mc;\quad \omega_p^2 = \frac{4\pi e^2 n_k}{m}\right)$$

(E is an elliptic integral).

Nonlinear oscillations of the electric field with amplitude $\varepsilon \sim \sqrt{8\pi n_\kappa W}$ and period T

$$T = \frac{4}{\omega_p\sqrt{\beta}} \mathrm{E}\left(\frac{\pi}{2};\ \sqrt{1-\beta^2}\right)$$

therefore occur.

Up to now the ionization of the Compton electrons has not been taken into account. However, the Compton electrons produce a large number of secondary electrons, which results in the appearance of a conduction current.

It can be shown that damping of the field because of the originating conductivity becomes substantial for a conduction-electron concentration n_e exceeding $n_* = n_k(\nu_{st}/\omega_p)$ (it is assumed that the collision frequency of the conduction electrons is $\nu_{st} \gg \omega_p$). Upon the achievement of such a concentration, the energy loss of the Compton electron is $\Delta W = \langle J\rangle \nu_{st}/\omega_p$ ($\langle J \rangle$ = 33 eV is the energy expended by the Compton electron in the formation of one conduction electron). For $W/\langle J\rangle \gg 1$ there exists a broad range of values of the parameters. of the problem for which the energy losses of the Compton electrons are small compared to W, but the damping of the electric field is substantial because of the originating conductivity.

The characteristic time of conduction-electron attachment to oxygen molecules is $\theta = [10^{-7}/(7.27\delta^2 + 2.3\delta)]$ sec (δ is the ratio between air density and the normal density) [5], and the time of electric field variation is $\tau_E \sim 1/\omega_p$. For the values of the parameters used $\frac{\tau_E}{\theta} \approx 0{,}23\left(\delta^{3/2} + 0{,}316\delta^{1/2}\right) \ll 1$, which permits neglecting the diminution in the conduction-electron concentration in the time intervals of interest to us.

Without taking account of electron attachment to the air molecules, the conduction-electron concentration satisfies the kinetic equation

$$\frac{dn_e}{d\tau} = \frac{\nu}{el}|j| + \alpha(T_e)\,n_e n_a, \qquad (2)$$

where $\nu \simeq 3\cdot 10^4$ is the number of electron—ion pairs being formed upon absorption of 1 MeV energy in air [1]; n_a is the air molecule concentration, $\alpha(T_e) = c_0 T_e\sqrt{\frac{8T_e}{\pi m}}\left(2 + \frac{I}{T_e}\sim\right)e^{-\frac{I}{T_e}}$ is

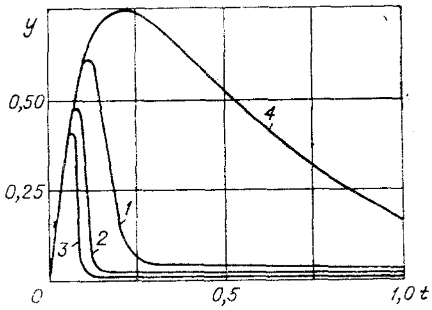

Fig. 1

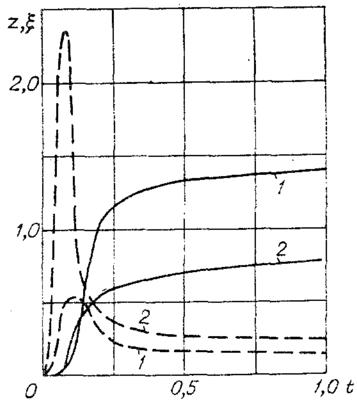

Fig. 2

the ionization rate constant [6] (I = 1.56 eV, c_0 = $0.85 \cdot 10^{-17}$ cm^2/eV), and T_e is the temperature of the conduction electrons. The energy-balance equation for the conduction electron is

$$\frac{d}{d\tau}\left(\frac{3}{2}n_e T_e\right) = \frac{e^2 n_e}{m\nu_{st}} E^2 - I\alpha(T_e) n_e n_a. \tag{3}$$

It was assumed in writing (2) that the initial energy of those conduction electrons which are generated by the Compton electrons is small compared to the energy they take away from the field.

Equations (2) and (3) are solved jointly with the Maxwell equation, which becomes (if the conduction current is taken into account therein)

$$\frac{dE}{d\tau} = -4\pi\left\{j(\Phi) + \frac{e^2 n_e}{m\nu_{st}} E\right\}. \tag{4}$$

The constant coefficients in the system (2)-(4) depend in a complex manner on the fundamental parameters of the problem, the total number of gamma-quanta, the degree of rarefaction of the air, and the distance from the source. However, if we introduce the following dimensionless variables;

$$t = \tau\omega_p;\ x = \frac{e\Phi}{p_0};\ y = \frac{E}{\sqrt{8\pi n_h W}};\ z = \frac{n_e}{\nu n_h};\ \xi = \frac{T_e}{I},$$

then this dependence will be determined by a single dimensionless parameter,

$$A = \sqrt{\frac{2W}{I}\frac{\omega_p^2}{\nu_{st}\nu_i}} \quad (\nu_i = \alpha(I) n_a).$$

The dimensionless system of equations (2)-(4) was solved numerically for different values of the parameter A. The results of the computation are represented in Figs. 1 and 2. The dependence of the dimensionless value of the field y on the time t is shown in Fig. 1. (the curve 1 is for A = 1; 2 is for A = 10; 3 is for A = 100). Presented here for comparison is a curve of the time variation of the electric field without taking account of field heating of the conduction electrons (curve 4). Figure 2 illustrates the change in conduction-electron concentration z (solid curves) and temperature ξ (dashes) for two values of the parameter A [1) A = 1; 2) A = 10]. Computations show that an avalanche increase in the conduction electrons occurs for values A = 1-100 of the parameter because of their being heated by the electric field. This results in a substantial narrowing of the electric field pulse width in comparison to the characteristic time of field variation without taking account of this effect (curve 4 in Fig. 1).

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